FOOTNOTES ON THE FOUNDATIONS OF GAME THEORY

F2. Revolutionary Dynamics

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The reader should be informed I have been hacked, with overt political purpose, and is encouraged to interpret results with the usual caution. Although I have done my best to report accurate results, confirmed interference of the sadistic kind has sabotaged some of my efforts. This involves manipulation of the interface – including expanding documents without my consent; ironic manipulation of results induced to the contrary of reason; rigged crashes etc. Notwithstanding, I recommend anyone interested in the subject to read, as the value of these general ideas is not sensitive to the reliability of a few data points. Succumbing to intimidation of this kind only makes you worse off anyway.

Surveillance and analogue harassment is used to accentuate the intent of illicit interference. It involves offensive behaviour on the streets, like quoting the latest parts of my work before it has been published; or while I am writing. Even persistent synchronisation with my work-habits is part of the oppressive efforts.

I am currently working from Malmö, Sweden, and hope colleagues around the world are exempt from such repression, which can only come about with the consent of authorities. This connection is among other things exemplified by the fact that I am writing from a computer which should be offline; and the overall illicit surveillance and sabotage of the kind I have suffered, is just too costly and technologically demanding, for other entities to carry out for prolonged periods of time without support.

These vulgar attempts to assert dominance have proven to be without any effect on my intellectual integrity over the years, my work continues to be independent, and I remain completely unshaken in my conviction of the significance of free independent thought. I am unimpressed by these efforts, as the only realistic prospect of such inexcusable actions is the ruin of the wrongdoers and their accomplices, unless those afflicted choose inaction and prefer the actual execution of threats in silence.

Finally, there is a comical aspect of this treatment which deserves commentary, as some of these jokers evidently display a curious habit of posing as intellectual superiors. Comments on the ideological and colonial underpinnings of such a ludicrous attitude are redundant, as they are obvious. But it takes a remarkable mindset to a) snoop on researchers and then b) pose as a genius with sincere conviction, and finally c) make afflicted researchers aware of this attitude and illicit snooping on their work. It takes an incredible, almost supernatural amount of denial to act superior with such scheme. The common man would just infer that it amounted to self-refutation, considering the source. It would appear to the mere mortal that chauvinism based on this fraud could not possibly inspire genuine confidence in sane men; nor could it incite genuine sense of inferiority on their targets as it amounts to sincere recognition. I am not flattered.

Manuel Echeverría, 2024-09-10

ABSTRACT

This paper provides a formal framework to advance heterodox alternatives to mainstream Economics by enabling syntheses with Game Theory. The arguments are carried out with an essential formula (**E0**) on the possible states in games with random strategies; or three proofs on how arbitrary games are generated and computed. In this manner, **Revolutionary Dynamics**, a perspective on how equilibria are disrupted and transformed is articulated. **Proposition 1** (P1) on how solvable systems are decomposed serves as a link to **Chain-Conjecture, cc[]**, which makes use of solution distributions from solved problems to deal with harder; and **Game Generation Principe (GGP)**, which allows **gmix[]**, universal solver for mixed strategies, to compute large games from minor ones stepwise. **E0** is a fundamental formula, underpinning **gmix[]**, with radical implications on the nature of society. It provides a threshold for fundamental uncertainty, *on arbitrary finite games*, which will persist under perfect information, even in settings with just a few independent actors with a handful of strategies.

The second part explores theory's potential with reference to **Systems Dynamics** and **Dialectical Materialism**; the social character of Universal Solvers; and how societies respond to fundamental uncertainty.

Keywords: Applied Mathematics; Economics; Game Theory; Philosophy.

UNIVERSAL SOLVERS

An independent perspective on dynamics is articulated with three proofs. First part of the paper begins with a mathematical expression of the main obstacles, namely the number of problems the solver must deal with. Then the construction project to overcome them is discussed in terms of Game Theory. This is not solely due to purposes of exposition and pedagogical concerns. Terminology hints construction can be seen as a game; certain design choices as strategies; the final construct as an outcome, even as a best-reply solution; with mathematical properties to be expressed as formulas, etc. Understanding also is advanced with perspectives from Swarm Optimisation and Graph theory. This discussion culminates in a solution concept to compute larger games, intimately linked with the proofs underpinning Revolutionary Dynamics – a view which disrupts orthodox notions concerned with perturbations and oscillations around particular equilibria. It provides a method to examine how equilibria transform, which serves the examination of why.

The second part explores the possibilities of this approach, its relation to *mainstream Economics*, *System Dynamics*, and *Dialectical Materialism*.

As the name implies, **gmix[]** finds all mixed-strategy equilibria and pure-strategy solutions. However, the latter are readily computed with **g[]**, *which is up to 1100 times faster* than claims about alternative designs, and manages hundreds of thousands of states with ease. It was presented in the previous paper *On the Foundations of Game Theory*.

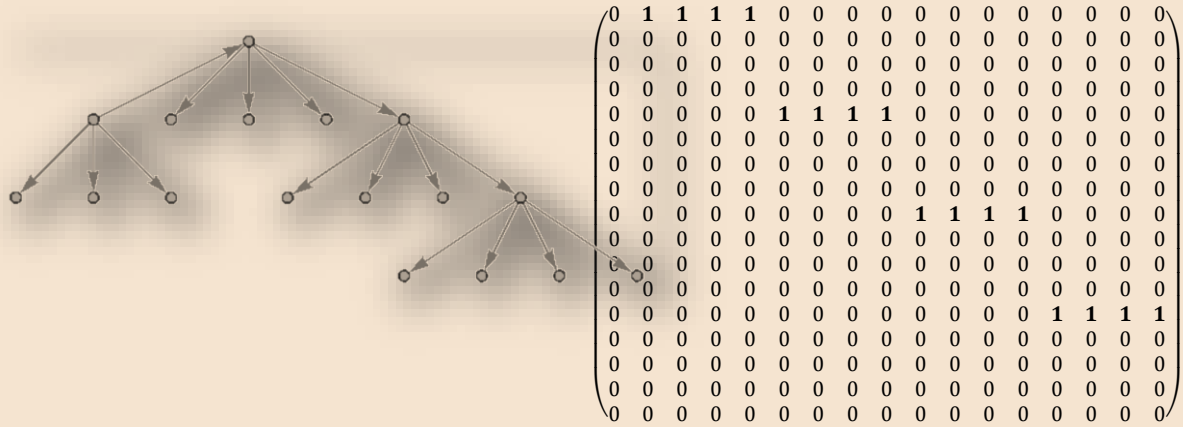
This is a research, pedagogy and training project. The latter requires a brief explanation. In short, no literature is used to construct Universal Solvers, beyond what is learnt within the scope of a few basic lectures. This provides intense training in self-criticism, deduction, methodical work flow with documentation, proofs, familiarity with generalisation from the simplest of cases, problem solving, and divergent thinking. The reader will not be bored with details but I recommend it, as overall scientific aptitude improves.

PART 1

MATHEMATICAL THEORY OF UNIVERAL SOLVERS

A mathematical theory of Universal Solvers is interesting in itself, but has tangible applications, as it will be able to predict the behaviour of **g[]** and **gmix[]**. Although the following example is simple, it makes a general point. In important cases and regards, *departures from pure mathematics can be seen as excess cost*. For instance, suppose one wants to generate a directed graph (game tree), where each player has the same number of choices.

F0. Simple Example: nodes[4,4]



Such graph can be represented by an adjacency matrix generated by an algorithm, say **nodes[**#players, #strategies**]** in functional or sequential programming without loops. Nevertheless, the following recursive pattern emerges. If nr. zeroes and choices are z_k and c_k respectively, then: $z_0 = 1$; $z_1 = c_0 + z_0$, $z_2 = c_1 + z_1$, $z_3 = c_2 + c_2 + z_2$, and $z_n = \sum_1^n c_i + z_{n-1}$ =/const. & sym. choices/= $cn + z_{n-1}$. This differential equation can be solved for $1 + cn(1+n)/2$ for numbers of zeroes before ones at each player row, which makes it easy to compute the rest by just adding n-1 rows of zeroes.

Because this formula can predict any adjacency matrix of aforementioned class, it can be used to construct matrices directly; navigate, effectively serving as indexation; and to create theorems about **nodes[]**.

Universal solver **gmix[]** is full of such patterns to be harnessed. Deduction of the combinations of mixed-strategy play (less full mix) is fundamental:

E0.Indexation Formula

$$\sum_{k=1}^{\bar{S}} \prod_{i \in I_k}^{\oplus} \left(\sum_{j=1}^{\bar{s}_i} \binom{\bar{s}_i}{j} - 1 \right)$$

In this context**, S** is the set of strategy sets $\mathbf{s_i}$ for each player **i**. The terms of the parenthesis are denoted $|s_i|$, and overscript denotes the number of elements. The *Subset-Product-Sum Operator* (SPS)

$$\prod_{i \in I_k}^{\oplus}$$

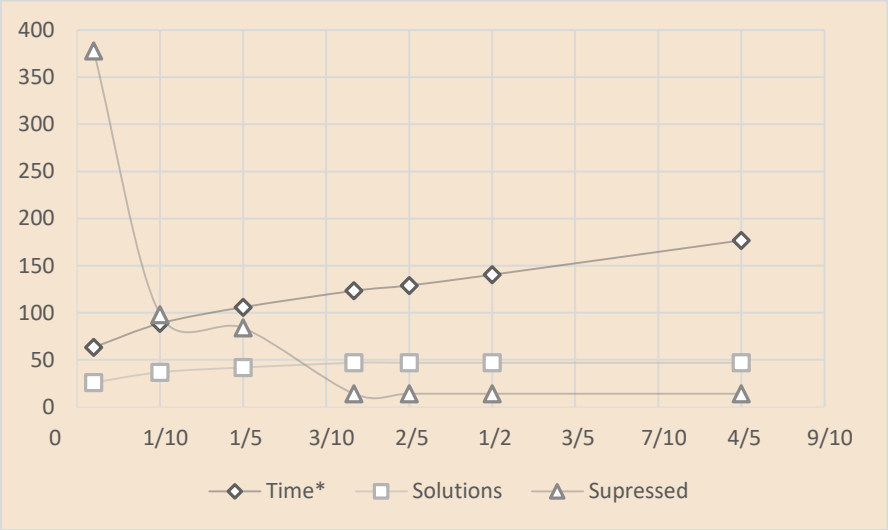
acts on a set **S** and returns sums of products (σ), corresponding to its subsets of size z, by exhausting all combinations of products of subsets of size z = 1, 2, 3,…, $\bar{S}$. At I_1, the subsets to be summed are of size one: $\sigma_1=\{|s_1|,|s_2|,\ldots|s_n|\}$; At I_2, $\sigma_2=\{|s_1||s_2|,|s_1||s_3|,|s_2||s_3|,\ldots,|s_{n-1}||s_n|\}$. The last product set is $\sigma_n=\{|s_1||s_2|s_3|\cdot\ldots\cdot|s_n|\}$, thus SPS returns the sum of a single term. *The expression E0 is due to the tuples of all sets of possible mixed strategies, combined with all possible subsets of players*.

This general compact approach is one way of improving code, but comes with limitations. Formulas like E0, and alternative articulations, may be irreducible, still cumbersome to compute in practice. Otherwise, such expression alone could replace sequences in the billions. Just the term $|s_i|$ is enough to infer that a 2-player game with 20x20 to 25x25 strategies is the upper bound *with procedures computing all combinations in memory* (benchmark). From there, games explode with sets in the billions, which ends up exceeding memory constraints with normal hardware. As the second part of this paper makes clear, this formula has radical fundamental implications on the nature of societies.

HARD PROBLEMS

Many Game-theoretical problems are known to be hard. Thus, themes such as reducibility, computability, and time constraints are an integral part of Game Theory. Figure 1 encapsulates the variables at play when dealing with *mixed strategies*[1] for a seven-player game: time spent (CPU+display), number of solutions, and cancelled ones due to breach of time constraint.

F1. Stylised facts: Single Timer & Solutions



This 128-state game has a robust pattern for budget breaches. The more players randomise, the slower when all players have the same amount of strategies. Naturally, the number of supressed drops sharply when time budget is relaxed: From hundreds omitted when the timer is at t=1/50, with 26 solutions found; to 14 omitted when all but one randomise, at t=1/3, with 47 solutions. However, moving beyond cubic forms to 6^{th} order or more, equation crunching takes too much time for generic equation solvers, and such equations are just but one component of a much larger system. Methods from Swarm Optimisation and Graph Theory are considered to solve the hard ones.

[1] Currently much faster, but overall picture is representative. Note that pure-strategy settings are solved instantly with **g[]**, unless several 100k states.

GAME THEORY FORMULATION

Application of mathematical Economics, Evolutionary Game Theory, and associated disciplines, was negligible when constructing the solvers, for all practical purposes, but useful contemplating design in the abstract. This section frames the subject accordingly, bridging to known results. In summary, the time-consumption issue boils down to a utility maximisation problem. This means that the algorithm can be assumed to enjoy solving problems successfully, but dislikes spending time doing so. Topics such as perfect information about problems, differs from incomplete information; how this feeds into design considerations; and how such designs can be conceived etc., are readily grasped when formulated as such.

The first is how much effort to spend on problems, avoiding spending time in vain when failing within time constraints. Thus, a probability distribution for timers, given a known distribution of problem-crunching times, must be contemplated. F_1 is a cumulative density of the former, F_2 the latter. T is the random variable of computation time, with T = t as a realisation; where $T_1 = t_1$ can be regarded as realisation of effort, and $T_1 = t_2$ of problem difficulty. The probability a problem gets solved is:

E1. $F_1(T_1 \geq t_2)F(T_2 \leq t_2)$ iff $P(T_1 \geq t_2)=[1- F_1(T_1 \leq t_2)]F_2(T_2 \leq t_2)=P(t_1,t_2)$

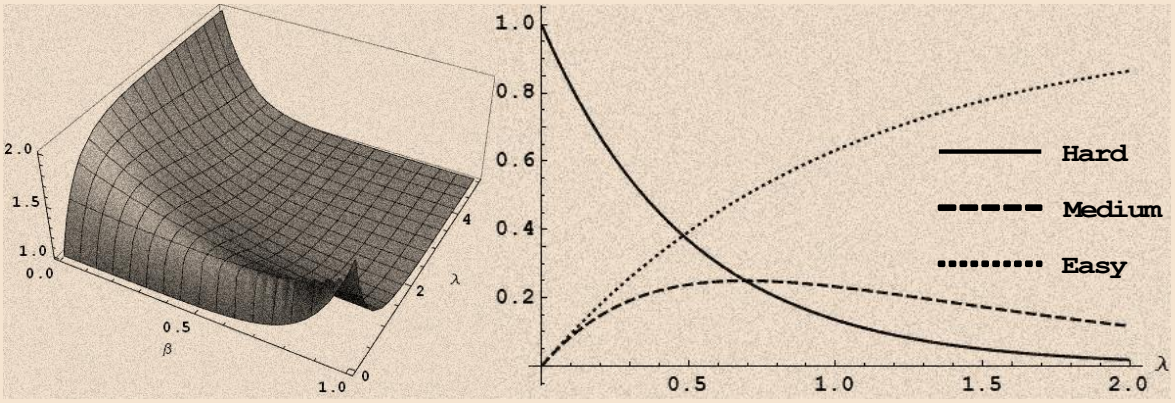
This is true for one problem. The joint of solving a series of problems is

$\forall i P(T_{1i} \geq t_{2i}) = \prod_{i=1}^{n} P(t_{1i}, t_{2i})$, where the index i denotes problem i={1,2,…n} in the sequence. The probability problem-solving lasts $\bar{t}$ is $P(T_1 \geq \bar{t}) = [1- F_1(T_1 \leq \bar{t})]F_2(T_2 \leq \bar{t}) = P(T_1, \bar{t})$, as solver halts when problem is solved.

In a slightly different setting, the tension between solving problems without getting stuck with hard ones can be expressed as preferences for certain intervals of outcomes. One example is binary relations $R(t, \bar{t}_i)$, over some interval of the CDF, with associated probability F_i, and preferences given by the value of $G_i(.)$. For purpose of exposition, now F_i *can be taken to reflect* probability of solving problems, a process which comes with benefits of success, but also costs in terms of processing time regardless.

In particular relations partition the set of outcomes s.t. $\sum_{i=1}^{n} F_i[R(t,\bar{t}_i)] = 1$, but indeed not necessary. This amounts to $\sum_{i=1}^{n} G_i[F_i[R(t,\bar{t}_i)]]$.

F2. Example: $\sum_{i=1}^{n} F_i[T \geq \bar{t}_i]^{\beta_i}, \sum_{i=1}^{n} \beta_i = 1, \bar{t}_1 > \bar{t}_2 > \ldots > \bar{t}_n$



Left:i=2, Right: i=3

Figure 2 displays preferences over two distinct outcomes to the left, and three to the right. $\lambda > 0$ shapes a distribution taken to reflect effort spent on problems. Higher values makes less effort more likely. If the more demanding tasks are strictly preferred, i.e. high β_1, then a low λ commonly is preferred. The value function is on the other hand maximised with a high λ when the converse is true.

Consider a more general mechanism requiring success-likelihood targets are met for some interval of the joint distribution of effort and problem difficulty, given certain local or global time budgets are met (spec.1 & spec.2). In this case, the joint distribution is a product due to independence. Effort-choice is denoted $\mathbf{T_e}$, problem difficulty $\mathbf{T_p}$, intervals I_k of magnitude/number of observations $|I_k|$, which can be deduced form **E0**. $T_e[\boldsymbol{\lambda}]$ emphasises parameters $\boldsymbol{\lambda}$ of the marginal distribution of T_e. Effort Distribution is chosen via the controls $\boldsymbol{\lambda}$ in the following system:

E2. A Distribution Mechanism

$$\bigcup_{k=1}^{n} P[T_e[\boldsymbol{\lambda}] \geq T_p | T_p \in I_k] \geq \mathbf{spec.1} \text{ and } |I_k| E[T_p | T_p \in I_k] \leq \mathbf{spec.2}$$

In effect, the solver is carrying out a mixed-strategy concerning effort levels given information of the problem distribution, and requirements given by preferences. **E0** can be used to map out regions where more challenging problems reside, given degree of mix-difficulty connection. Clearly, optimisation may be a criteria, e.g. aim to minimise time spent given requirements.

In practice, this setup works remarkably well, with expected values close to observed, and straightforward to implement refinements to exploit features of the distributions along these lines (see Appendix).

Now, if the costs, i.e. necessary time to solve, were known for sure, it would be possible to minimise costs for each problem faced. If the algorithm knew exactly what to expect, the most economical thing to do would be to set $t_e = t_p$, as long as the incremental benefit of doing so is positive. When even less is known, it is instructive to turn to Game Theory.

For purpose of exposition, consider a Game-Theoretical setting where the story is a War-of-Attrition Game (see e.g. Webb 2007)[2]. This usually is a chicken race with net-benefit-maximising players, fighting to control a beneficial resource by incurring a cost, like effort. In this section however, the opponent is a distribution of problems to be solved, generated by circumstances such as default mathematical procedure to solve a problem; the nature of the problem; how the problem grows with increased interaction (e.g. nr. of players) etc., and additional unknown factors.

The Game-Theoretical solution revolves around a standard, but in this context useful, expected value or utility calculation. Consider a utility function $S(\mathbf{E}, \mathbf{C})$ with probability distributions as arguments. Here $\mathbf{C}(c)$ is the difficulty distribution of problems or *cost density* of *required* time $c(r) \in \mathbb{R}^+$, where r is supressed. Likewise, $\mathbf{E(e)}$ is in general a distribution of effort, i.e. a mixed strategy.

[2] Great book overall but I have some reservations, as presented here.

If the algorithm plans to spend more time on a problem than required to solve, it gets benefit b minus time t = r required to solve it, but is just left with effort **e**(t) spent in vain if it spends less than required. Therefore, given effort just is time spent (e = t), the expected utility of the algorithm is:

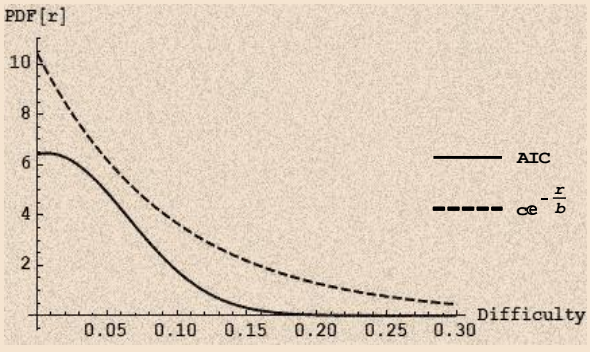
E3. Algorithm's Utility Function & Expected Utility

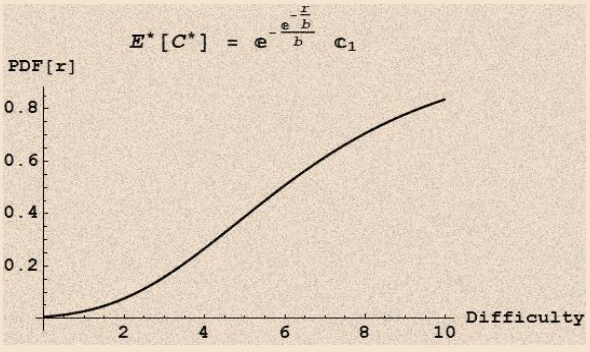
$$s(e,r)=\begin{cases} b-r & \text{if } e>r \\ -e & \text{if } e<r \end{cases} \qquad \int_0^t (b-r)c[r]dr - \int_t^\infty tc[r]dr$$

This yields a decision function for the problems, given a level of effort of the algorithm $C^*(e)$. Likewise, using problem perspective to attain a rule for the algorithm, yields effort spent *given problem difficulty*, i.e. $E^*(c)$, depending on notation, where stars denote optimal levels. Such view is allowed via the Fundamental Theorem as variables are swapped, from *r* to *t* in this case. A minor philosophical digression is warranted as a heuristic.

Note that $C^*[t]$ can be substituted with $C^*[r]$, as if their arguments were conventions. The same is true for $E^*[r]$ and $E^*[t]$. Moreover, in this example, $C^*[]$ and $E^*[]$ coincide in form. Optimal effort can also just be seen as a function of time *t*, i.e. E[t], just a persistence rule. In effect, nothing hinders computation of expected value with an effort function once the solutions are attained in this example. Decision rules, i.e. statistical distributions, for problem difficulty and effort spent are exponential in their corresponding arguments. In this case, however, when solved as $E^*(C^*)$, time spent attains a logistic form.

F3. Solution & Heuristics





From differential equations $c_1 = 1/b$ can be derived with the requirement that the total probability mass equals 1. AIC and $b \approx 0.96$ from estimation, which also are relevant options for E2.

In practice, the decision rule to the left is plausible when thousands of problems are to be solved. Attempting them all with a stubborn algorithm amounts to spending too much time in vain. Higher benefits imply longer tails, i.e. it is worth taking on more challenging tasks, as mean effort is uniquely determined by *b*. A more elaborated cost function would just serve as a countervailing incentive in the nominator of the exponential. The approach also is relevant when problems may get harder in order to beat the algorithm. Although security reasons are salient, a kind of equilibrium condition where capacity of the computer is under strain can be considered.

The second heuristic rule is not as counterintuitive as it may seem at first glance, but it is more difficult to check for Kolmogorov conditions. When the relative payoff of the algorithm is miniscule relative to the 'payoff of the problem', then the probability increases much slower.

High-powered incentives i.e. a scheme on effort spent yields distributions related to the exponential but with time as basis, and such can be made to satisfy the conditions for a probability distribution proper.

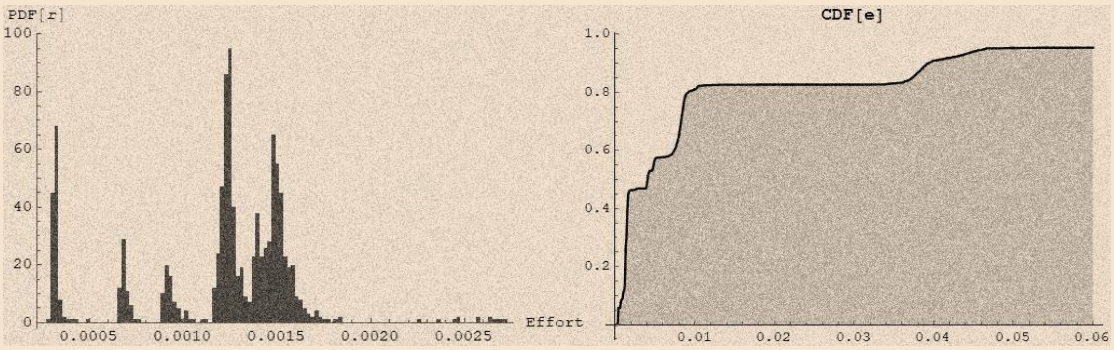
MULTIPLE TIMES

The following rather informal discussion pinpoints that more elaborated models of the time-management problem are redundant for now, as the major conceptual issues can be grasped and coded without. Data, in terms of the empirical distribution has clear instances roughly corresponding exponential distributions, but largely when zoomed into dense areas. The multimodal graph to the left exemplifies this at the scale 1/1000.

The cumulative density shows that nearly all weight (95%) is at $r \leq 6/100$. However, these barely make up 9% of total time.

70 % of time spent is on the 14 last problems (~7/1000). These facts emphasise that more intricate decision rules as outlined above may not deal with the core problem.

F4.Empirical Distribution of Effort (Double-Timer Design)



The following backdrop is relevant for huge systems to be solved, e.g. to attain Nash Equilibria. Interaction between technology and problems amounts to games which can be can be one-shot, dynamic, repeated, with incomplete information, etc. Consider a setting where technology at hand is a slower generic analytical solver which yields high-quality expressions for exact solutions; and a faster generic numerical with comparably lower-quality approximations. Problems either have analytical solutions and easy; analytical and hard; or cannot be solved at all analytically, or in reasonable time, hence classified as inherently numerical. In what follows, it will become apparent that 'solvers' can be replaced by 'solutions'.

T1. Technology and Problem-Type Interaction

Solvers\Problems	Easy Analytical	Hard Analytical	Numerical
Analytical	($\mathbf{e_{11}}$, **r1**)	($-\mathbf{e_{12}}$, r2)	($-\mathbf{e_{13}}$, r3)
Numerical	(e_{21}, r1)	($\mathbf{e_{22}}$, **r2**)	($\mathbf{e_{23}}$, **r3**)

The *outcomes*, starting from the first row to the left are: *Fast & High quality; Slow & High quality; Nothing; Faster & Lower quality; Faster & Lower quality; Slow & Lower quality*.

The following preferences are reasonable with Universal Solvers. When time is a consideration, but fast high quality is still better than slightly faster but lower quality, then the analytical solver is preferred. Otherwise the numerical is preferred until problems are solved or time is up. Symptomatically, when payoffs e_{11} and e_{23} are greatest, a randomising equilibrium emerges, in addition to the pure-strategy equilibria underlined.

If only one timer was used, time-consumption would hike with an analytical solver and a generous time budget, and it could still be in vain. But many problems could possibly remain unsolved with a stricter budget. The conundrum is to allocate effort between solvers, which requires knowledge of the underlying distribution or guesses as (Algorithm) War of Attrition. To deal with this, a multiple-timer mechanism is a salient approach.

In the simplest of cases two are used: a timer is set for analytical solutions; and if the problem is not solved within time, the program switches to a faster numerical one, suppressing problems which take too long. An additional interesting enough trade-off emerges, characterised by risk of inefficiently low time constraints on analytical solutions, resulting in numerical retakes, perhaps without getting the solution in the end. Different combinations of tech imply different games for the same set of problems in many cases, vice versa. Adaptation over time is a possibility and necessary in series of new games differing in size and composition, and the solvers themselves could undergo evolution to the game-theoretical problems at hand. That is the topic for the next section.

CONCLUSIONS 1

Timer solutions are but a special case of a more general aim to exploit the underlying problem-difficulty distribution, but quite enough to improve things drastically. Considering algorithms and problem solving as games proves useful to evaluate and develop universal-solver designing. More elaborated mechanism design is an obvious extension to what has been

discussed, as it would provide the possibility of optimal incentive schemes given information about the distribution to adaptive agents. However, the basic theoretical points regarding time management when facing distributions of difficult problems, would not be radically altered. Notwithstanding, a global time budget and allocation of time between solvers adjusted to the character of the problems improves things drastically and is aided by information of the difficulty distribution.

REMAINING HARD CORE

Hard problems generic solvers fail to overcome within reasonable time are in need of special attention. In terms of the 128-state setting, the remaining one is a fully-mixed problem, which generic solvers fail to compute within hours. This is reduced to a fraction of a second with a network approach with a basic numerical routine. This is but one reason to take a more general perspective on the underlying systems of equations which pinpoints networks, followed by instructive proofs. This understanding goes beyond systems in universal solvers presented here.

In what follows, *a system* is a set of conditions/ equations; and a solution is an assignment of values to each variable, consistent with all conditions within the system. A *component* is a subset of these conditions, *which can be satisfied by such solution*, *and this property partitions the set*. For instance, **E0** underpins such partition, as it tracks degree of mix, e.g. in systems of incentives.

The basic but tremendously useful point here is that although a generic algorithm may take its time trying to solve the whole system, it is still possible to check a subset of a tardy system partially, and then check if each of that component’s solution is consistent with the whole system. If there is system-wide solution, then it must satisfy all components.

Among other things, this amounts to using bits of information about easier components, in order to solve the more challenging enigma. A related question is whether exhausting the list of partial solutions, without finding the global, really implies that no solution exists. The other thing worth contemplating is when solution candidates of each component are close. If there is a non-empty full conjunction from the set of all solutions of each component, i.e. if a given solution appears at each partition/component, then it is a system-wide solution. This sets the stage for a Swarm-Optimisation or network approach which considers clusters of similar solutions that convey key information about others.

DECOMPOSITION PROPOSITION

The following proposition is a succinct way of expressing aforementioned. The following conventions are used. $\mathbf{x_i^*}$ is a solution to a system $S_i(x_i)=\{c_1(x_{i1}),\ldots,c_2(x_{ij}),\ldots,c_n(x_{in})\}$, when $S(\mathbf{x_i^*})=\{\mathbf{0}\}$, i.e. all components are satisfied, *denoted* $c_i(\mathbf{x_{ij}^*})=0$, and there exists an exhaustive list of solutions $\mathbf{X_i^*}$ s.t. $\mathbf{x_i^*} \in \mathbf{X_i^*}$ for system S_i, and the values of the solutions satisfy a set of restrictions **X**, denoted $\mathbf{X_i^*} \mathring{\in} \mathbf{X}$, where $\mathring{\in}$ retrieves variables on LHS, and sets them as elements of corresponding RHS entries, forming a restriction set.

Definition: A system is *trivial* if each or some components can be solved independently of the other. For non-trivial systems, the following is true.

Proposition 1. There exists a solution iff. $\forall\, \mathbf{x_{ij}^*} \in \mathbf{x_i^*} \cap_j \mathbf{x_{ij}^*} \neq \emptyset$, s.t. $\mathbf{X_i^*} \mathring{\in} \mathbf{X}$

Proof. The RHS implication is true by definition. Assume $\forall \mathbf{x_{ij}^*} \in \mathbf{x_i^*} \cap_i \mathbf{x_{ij}^*} \neq \emptyset$**,** but not $S(\mathbf{x_i^*})=\{\mathbf{0}\}$. By definition, $S(\mathbf{x_i^*})=\{\mathbf{0}\}$ and $S(\mathbf{x_i^*})\neq\{\mathbf{0}\}$, which is a contradiction. From LHS: If $\forall \mathbf{x_{ij}^*} \in \mathbf{x_i^*} \cap_j \mathbf{x_{ij}^*}=\emptyset$ and $S(\mathbf{x_i^*})=\{\mathbf{0}\}$, then the system is *trivial* if the intersection is true for each component. $\cap_j \mathbf{x_{ij}^*}=\emptyset$ and $\cap_{j\notin T} \mathbf{x_{ij}^*} \neq \emptyset$ is possible. But then $c_k(\mathbf{x_{kj}^*}) = \mathbf{0}\ \ \forall k \in T$ without connection, i.e. trivial systems with the connected others is the case.

The proposition is enough to proceed. It is possible to solve each component separately, and look for the intersection across components – exact by an analytical solver; or approximately for numerical. If a value contradicts proposition 1, it cannot be a non-trivial solution for that system.

Remark. A core assumption of Proposition 1 is a list of solution candidates, but such solutions may not be very useful unless processed e.g. if the system has underdetermined components. A single equation also is a component. A component is a system of equations, but called the former in a system of components. Because the system deals with sets, the trivial system of several equal components is ruled out. Also note that the restriction **X** could have been indexed and contain less than all variables. *The intersection is on values assigned to variables*.

THE NETWORK SOLUTION – CHAIN DIVINATION

This section departs from the stylised facts mentioned in the introduction, and in view of Figure 1 to pinpoint a hard problem. There is a statistically and theoretically robust and clear-enough pattern of a spectrum ranging from *easy-pure-fast* to *hard-mixed-slow* to be harnessed. While Universal Solver **g[]**, handles pure-strategy games with several 100k states; the fully-mixed component of the 2^7 problem was not even computed in lunch time with standard algorithms. The theoretical rationale is simple enough to grasp. Whenever players use pure strategies the system is simplified, as the number of unknown variables to be solved is reduced.

Proposition 1 also relates to how a network of components may aid cracking unsolved problems. Starting with a graph **G[]** of problems linked to mixed strategies, there exists a sub-graph $\mathbf{G[}c_j, \mathbf{x}_j^*\mathbf{]}$ of problems linked to solutions. Such would typically amount to a couple of stars amidst pairs of connections. There is a corresponding graph $\mathbf{G[x}_{jk}^*\mathbf{]}$ of solutions, linked if they have the same amount of discarded strategies, which is related to difficulty in various applications.

The former graph gets richer when problems are allowed to be linked by successful permutations of one or several solutions. In essence, chain divination generates such graph.

To give the proposed solution a context, note solved problems may provide information about likely solutions of immediate neighbours, and can thus be used to guess linked unsolved problems. Centrality of specific solutions could then e.g. be connected to concepts of local and global.

Underlying guessing pattern is naturally linked, but not limited to, Swarm Optimisation methods[3] as solved problems can be perceived as a population adapting environments with multiple optima, to crack unsolved ones. A salient alternative formulation is solutions linking nodes of problems, e.g. components, connected by solutions.

A special case is to take one immediate neighbour, replace its discarded (pure-strategy) elements with a random number, and make it a guess. If not the answer, repeat[4]. The remaining fully-mixed problem which failed to be computed within lunchtime took merely 1/60 of a second with chain divination when employing immediate neighbours.

Beyond issues of commensurability between methods to solve equations, there is a conceptual issue lurking, highlighting the merits of paper 1. *Due to the random element, the number of solutions found within seconds may not exhaust the theoretical number*. Several options exist. Repeated samples can ameliorate without complicating the approach, and two additional solutions are indeed robust. Nevertheless, deriving the likelihood of the number of solutions, given dimensions and statistical distribution generating the game, in line with Echeverría (2023), would provide necessary information to know if all have been found for sure. Likewise, statistical simulation and replicator dynamics along these lines could aid a characterisation of the number of solutions.

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[3] For an excellent review, see e.g. Khemka & Jacob (2009).
[4] A special case of Swarm Optimisation, but the overarching framework is not.

The obvious generalisation is to make procedure's replacement distribution contingent on patterns found among neighbours, e.g. whether a one or zero is replaced. More importantly, use the set of clusters to for chain divination at each level of mixing in the graph. Ultimately, such arrangement provides data to AI in order to discover new laws.

CHAIN DIVINATION

Consider the game **G** with an arbitrary number of players and strategies, each; but only a certain number are allowed to play, and these are restricted to specific strategies, as if obeying a set of higher rules; with arbitrary distributions determining incentives at each outcome after the aforementioned is settled.

Neither number of players nor strategies are a problem to control the scene. It is enough to invoke a list of players where all are removed or restricted to simpletons, except those allowed. Likewise, strategy sets can be restricted or set to zero. Then, an arbitrary incentive structure for outcomes can be chosen. Thus, a small game is just a restricted larger one; and the former can generate a larger one stepwise, e.g. one strategy at a time. For a given incentive distribution of expected values, a zeroed and non-existent strategy is the same. A conjecture from a mixer share of 1-1/N to a mixer share of 1, when all N strategies are mixed, has already been established (2^7 setting). In general, it is possible to expand the game from N to N+1 strategies, and use Nash Equilibria from N to guess problems at N+1. By Nash (1950), a solution exist at each step.

Remark. Works as good or bad at any step in principle, but guesses tend to get harder with more variables, for any equation-solver algorithm. But it is not by virtue of some quirk of game generation beyond comprehension.

Game Generation Principle and the Chain Conjectures just described. Technically it is possible to consider 'games' with just one simpleton player stuck with one strategy.

Definition. A minimal game is played by two simpletons. A full game is an arbitrary finite game (could be computed by **gmix[]**).

Lemma 1 (Game Generation Principle). Any game can be reduced to a minimal game, and any minimal game can be expanded to a full game with arbitrary number of players or strategies or both, generated with arbitrary distributions (not exclusively players'), as long as the game stays finite.

Lemma 1+. Chain Conjectures **cc[]**, can be used on arbitrary finite games

Proof Lemma 1. Given p players with strategies sets s_i, the state vector **S** is the Kronecker product of all the strategy sets, with dimension $n_s=\prod_{i=1}^{n}|s_i|$. The joint distribution **P** over states is constructed in the same manner but with corresponding probability distribution $\mathbf{p_i}$, i.e. probabilities over each state for each player, hence the dimension is the same.

A game is played over the outcomes for each player for each state, represented by a matrix **I** with dimensions $n_s$xp. Consequently, the expected value for each player given an arbitrary distribution (influenced by player choice) is the product of the probability of a state times the incentive for that state, and this is contained in **v** = **PI** with dimensions 1xp. Hence, only the dimension of players remains in the values underpinning the game, and this is what all further decisions are based on, which will determine the final outcome of the game. Given an arbitrary game **G** in this setting, it can be reworked as follows.

A joint distribution **P** can be altered to **P'** by changing an arbitrary player's distribution, say from $\mathbf{p_i}$ to $\mathbf{p'_i}$, and this can in particular be done by setting a player's probability of an arbitrary state to zero repeatedly, as long as it satisfies the Kolmogorov conditions. In this manner **P'** remains a joint

probability distribution, and because the dimension of **P'** remains unaltered, the corresponding expected values in **v'** are defined in each step, and so is **G'**. In particular, it can be done until $\mathbf{p_i}$ becomes a degenerate.

Because this is true for any arbitrary player i, another player j can be chosen given **P'** at any step. Thus a procedure with player i followed by j amounts to setting one of their strategies to zero each turn s.t. the conditions hold, and both the resulting distribution **p''** and **G''** are well-defined. But this is the same as doing the procedure simultaneously, by setting their probabilities to zero, and the argument can be iterated for any number of players until the minimal game is reached, which proves the first part. This argument can be used to reach any arbitrary game from a minimal by replacing an arbitrary zero with a probability variable at each step, and thus expanding the game. In particular, all the steps could be reversed to go back to **G**. This can be constructed e.g. by storing the sequence of zeroes at each step – or the full list if done simultaneously; or constructing suitable matrices instead.

Corollary. Universal Solver **gmix[]** can start from any given game, by restricting it to a smaller, and then proceed computing successively larger games, until reaching the unrestricted one, as allowed by GGP. But it can also to compute arbitrary ones created by generating inputs with arbitrary distributions repeatedly.

Proof Lemma 1+. The algorithm **cc[]** uses at least one Nash Equilibria (NE-bases) to generate vectors of mixed strategies to find a new one. By Lemma 1, a game can be reduce to a minimal one, and by Proposition 1, the game can be seen as a system of components which can be tested and solved. By Nash (1950), at least one equilibria exists at N=1. By Lemma 1, it can be increased with strategies to form an expanded game.

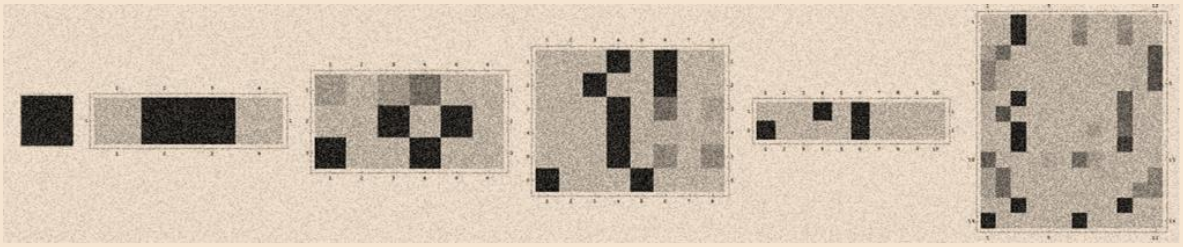
Assume **cc[]** has been applied to game N, then **cc[]** contains at least N NE-bases (degenerated or mixed), then it can applied to a N+1 system extended by Lemma 1, which proves the lemma by induction/construction.

REVOLUTIONARY DYNAMICS

This section introduces yet another way of invoking dynamics from so-called static games which, to my knowledge, appears to be overlooked, to the very least in comparison. Previously in the series, the reader was reminded of a well-known type of (replicator) dynamics, which adds more realism to Nash Equilibria, by addressing the somewhat mistaken criticism against supposedly perfectly rational players and static games.

With *Revolutionary Dynamics*, in line with Proposition 1 and GGP, equilibria themselves evolve. It goes beyond *Replicator Dynamics* as each alteration, allowed to differ in number of steps, can be of finite arbitrary dimensions and distributions, which upsets the underpinnings of equilibria at each alteration. While analysis of the latter focuses on perturbation of existing equilibria, given by a corresponding static game, *Revolutionary Dynamics* is about how interactions transform and why.

F5. Revolutionary Dynamics & Equilibria Transformation



Revolutionary Dynamics is a rigorous theory which allows the very equilibria to evolve in a general setting. For instance, starting from compact pure-strategies spaces, with all density concentrated to a few spots; and then expanding, as the game unfolds, with larger spaces occupied at random; and then once again contracting even in far more extended phases of the game. An analogy to a simple organism, gradually evolving, settling; then suddenly expanding and contracting as the situation transforms to a richer set of possibilities.

F5 presents the evolution of equilibria when the dimension of the game increases in steps of equal size, but the unfolding of the game neither presupposes steps of equilateral size, nor is restricted to the same generating distributions; and although expected values are computed, there are no demanding requirements on rationality. Some players can be stuck in the same game, others will see sequences or the whole.

Although Nash Equilibria can be computed, if capacity or time allows, not everyone needs to conform. Thus, instead of a fixed timeline, each alteration can be interpreted as an experiment or imagination of such.

The potential for applications in a wide range of fields is vast, some of which will be treated in what follows and in future papers, starting with what a universal solver says about the nature of theory and society.

CONCLUSIONS 2

Revolutionary Dynamics is a formal theory based on three proofs which outline the following method to solve larger games.

- **GGP** is used to reduce the game.
- **Proposition 1** is used to solve reduced games.
    - **cc[]** is employed, at least once an 'acceptable' number of solutions have been found.

The proofs make clear that arbitrary distributions can disrupt any given setting, allowing sequences of arbitrary games to be computed. Part 2 explores the possibilities in more detail.

PART 2

NATURE OF LIMITATIONS BEYOND TECHNOLOGY

Methods and technological matters have a natural correspondence to concepts in other disciplines, the social sciences in particular, as they underpin vital patterns of interaction; and our understanding of these – as such, artefacts for analogies to grasp the complexity of existence. But classes of explicit remarks of this kind usually are redundant here, as many of them should be familiar or obvious, in view of previous work. Instead of specific examples of particular people, events, category lists and associated relations; concepts furnished here are about how such strategies and interaction emerge, evolve, fit and breach general conceptual frameworks.

This paper begins a theoretical discussion on how such frameworks interact, and strikes a balance between construction and proof by contradiction. One of the reasons is that the former is instructive as an artefact in the aforementioned sense, but not always feasible. The latter can be efficient, but not always instructive and could in principle provide means to perpetuate ignorance about the underlying object of the proof. Partly, this can be employed to protect, but in view of the well-known effort threshold in formal sciences, and competition of societies, mystification and ideology could interact with strategies to assert hierarchy through the modes of dissemination of rational thought. Ideological systems slandering practical origins of theory reinforce these tendencies, by risking to perpetuate confusion accordingly; and one would be at fault to overlook the mundane incentives, along similar lines, from which such architecture of beliefs are erected. Thus, it is far from always ideas and opinions which guide relations and outcomes, but the technology which generates them, the method of distribution, and resource restrictions.

If there is one insight in this regard which can be singled out as prominent in Game Theory, it would be that although men can have different opinions on matters, what really matters is the bottom line; and that perfectly rational and irrational ideas of what happens may in the end lead to the same outcome, and the bearers of the latter can appear as wise as the

former by virtue of visceral necessity or measured accident. This curbs the domain of notions insisting on the primacy of ideas in human affairs in view of Evolutionary Game Theory, also underpinning hypotheses from the natural sciences.

NATURE OF LIMITATIONS

While pure-strategy solutions to the 2.5k game in paper 1 are computed instantly, the sun would literally fade in the process before all problems with impending mixed-strategy equilibria are solved with current settings. Both number of strategies and players are major factors above 20.

Indeed, there would hardly be resources to even compute fractions of all contingencies. If such undertaking was carried out regardless, algorithms would still have to be tailor made for specific games to exploit their specific structure, where removal of dominated strategies is but a minor part overall; or a general approach to account for the emergence of equilibria, as indicated in the first paper, could be venues for further progress.

To begin with, these limitations are considered as a measure of predictability challenges historical institutions have faced, even modern 20^{th}-century ones, and at least some prominent contemporary. Formula E0 underpins the partition of alternatives which makes possible computation of mixed-strategy equilibria. Such equilibria reflect a rational response to uncertainty in non-cooperative games, and preferred to pure-strategy alternatives in many relevant settings. As already stated, these can also be perceived as behaviour closer to everyday-notions of trial-and-error.

To imply, but not commit to a meaning; feint and let the enemy guess which attack is to be carried out next on an instant, is not just a matter of action in the moment. Fighters' pride of their particular style, is a tribute to the hyper-rationality tradition brings when there is little time to think. A hook forcing the opponent to react, making room for a perfectly placed uppercut which finishes the fight belongs to a set of sequences that have evolved as best replies within a particularly brutal mode of communication.

But when adversaries have studied their traditions, a possible best reply is to allow spontaneity, which is a creature lurking in the depths of instincts, but knows when to touch the surface to think clearly, adapt and vanish. Given limited mental faculties, this ability to functional reflection is in itself subject to constrained refinement. For concreteness, although a distribution over concepts could overcome a given problem, the ability to successfully consider distributions on a subset of a vast set of problems would be in need of such developments. The device under consideration pinpoints emergence of conventions or behaviour in settings which correspond to independent choices guided by self-interest. As such, it allows thought experiments in the vicinity between order and anarchy.

The conclusions derived from E0, is that a handful of players acting non-cooperatively is enough for organisations to face fundamental uncertainty, which could induce those in need of a strict chain of command to derail. The higher predictability of pure strategies is a disadvantage in many competitive settings, but its transparency is desirable with regard to control. However, this contrast is blurred with increasingly complex settings, as pure-strategy interaction will become intractable as well. Therefore, surveillance and complete information of system inputs is not sufficient to ensure predictability. The scope concerns any institution which can be described by arbitrary finite games, and this would remain true in societies which claim devotion to freedom.

These are some of the most fundamental possible, preceding analyses of particular plays, and consists only of the most primitive elements, by virtue of starting the discussion from universal solvers. It is one among several possible of theoretical interest. Notwithstanding, some elaborated notions of uncertainty or costs require additional assumptions, not even properly formalised in some cases; and simpler particular examples lack deeper connection to theory.

POLITICS OF SIMPLIFICATION

The construction of universal solvers is in itself a process to be considered as a game which embodies and exemplifies forces of production, social relations, institutions, an economy, and a politics of computation. This development was partly guided by responses to overcome the obstacles imposed by E0.

The trivial solution has staggering political implications on interaction in the conduct of human affairs. When an incentive structure is to be imposed, then restricting individuals to simpletons; rigging the game in terms of incentives; and only enabling a few to make choices of consequence, is not limited to immediate rewards or sadistic tendencies towards mental or physical mutilation, even when it is the whole purpose with the design, as in the case of some of the most gruesome practices across societies, throughout history, until present day. Ensuring predictability requires such schemes in certain settings, given technology at hand, e.g. to enable specialisation surrounding an automated work process. Even research environments are conceivable as ones where individuals carry out restricted tasks devoid of deeper thought, but enable an organisation endowed with universal computation to produce conceptual output. For configurations of institutions defining the continuity of a civilisation by means of gradual development, advanced technology is not a prerequisite; nor is the trivial solution invoking the humble reasoning of a single mind amidst stagnation necessary for such conjectures. Notwithstanding, the latter safeguards the proposition by bringing it closer to the set of tautologies.

In this regard, there is a fundamental remark to be made which transcends the division between schemes carried out once or repeatedly. It is about the ability of acting to meet the unexpected swiftly as a society, and this may require an entirely new game to be executed which upsets the conventions emerging in repeated ones. Such scenarios naturally emerge in conflict. Moreover, aforementioned manipulation of incentives can also prevent mixed strategies.

E0 implies an astonishing prediction. A handful of people with free will and meaningful alternatives, affecting final outcomes, is all it takes for a game to become intractable. It conveys what gives rise to demand for limiting decision-making through a machinery which works to separate a few decision-makers, with consequential actions, from the rest. It quantifies a conspicuously low threshold which ultimately links alienation to economics and power. Given an incentive structure, either the number of decisive options must be reduced; or the number of individuals with such options, or both. Hence, there are strong incentives to either divide society into spheres where individuals can take any action, as long as it does not interfere with the scheme under consideration; or induce obedience, regardless if in the self-interest of the subject by default or not.

As which one is to be restricted or removed is available to taste and choice, a politics of simplification emerges. In this manner, society is transformed towards games between Leviathans individuals are free to conform with, and these could be arranged to behave in accordance with cooperative or non-cooperative games. As a result, top-down perspectives become salient, i.e. a view of society as interacting components represented as functions or distributions which reflect the aggregated behaviour of individuals and the movement of variables characteristic of the machine. Such perspectives, formalised with Systems Dynamics, have at times been held as alternative ways of doing economics.

An intermediate response is to make the Game-Theoretical existence coarser by reducing randomness without excluding it. In technical terms, this just amounts to considering contingencies, in terms of problems, one or more players depart from randomised play with a distribution containing all mass on one option, i.e. a pure strategy, instead of discarding subsets of strategies in line with E0. Such device is less demanding and has its place in applications. It means that individuals may still conform to mixed

strategies, but will 'depart' resolutely.[5] Moreover, it becomes less demanding to allow groups of players to play, which makes the previous remark even more intuitive. However, this could imply an empty solution set, with far-reaching repercussions as it means that whereas simplification in terms limiting the freedom of players is feasible to induce order, limiting problem-solving once the game has been defined can induce chaos.

As it stands, there is some inherent waste in the sense that problems with highly mixed states can collapse to pure-strategy play as these can take any value satisfying the Kolmogorov conditions; but not the other way around. Although thought provoking, one should avoid superficial analogies, as thoughts may wander from simultaneous associations to a single, and conversely. This feat can lead to duplicates observed in 1/15-1/5 of the candidates in small games. This shares diminishes as games grow. Although not enough to address the E0-threshold, it provides incentives to understand the connection between neighbouring systems of equations where AI or advanced mathematics can play a meaningful role.

Randomising game generation is a response by extension of GGP that accounts for resource constraints. On the 'supply side', an extended GGP allows elementary objects of the game to be stored, like players, strategies, and values at each state. However, the evaluation of contingencies in decision-making, given actions of others, are constructed and retrieved to be solved at random. Moreover, the elementary objects can be generated by rules, and thus be stored as formulas and algorithms, like E0. This is to some extent how games in paper 1 were generated, but in this case, the random game is allowed, but not required, to follow a script. Parts constituting contingencies materialise at random with probabilities given by a Hypergeometrical distribution; and then the solution is registered.

[5] Symptomatically, such device was coded before current **gmix[]**, but does not coincide with **g[]**. Comparisons may appear in an update.

Furthermore, on the ‘demand side’, algorithms in line with Proposition 1 would have to work with systems of equations that are regarded as a component in view of E0, to the extent that is possible. However, this is a conjecture, which could prove to have limited scope.

EMERGENCE OF REVOLUTIONARY DYNAMICS

A world generated at random in view of resource constraints is a concept integral to society, and it makes sense to begin to grasp it by exploring the production process and the inner workings of universal solvers. In addition to aforementioned analogies to societies, these are devices to predict aspects of the interaction of the social world where they belong; and the choice of how to use or develop these or similar devices with available tools, are also decisions subject to strategic interaction; in particular to consider entities with such. Therefore, a critical level of connection to the social world is inherent in such a methodological choice from the outset, in a manner which invites multifaceted reflection. In view of Revolutionary Dynamics, and games constructed around hardware like the CPU, where the software corresponds to patterns of interaction and institutions, a scheme of progression about the emergence social relations takes form. In a small-scale beginning, new ones are readily formed by intent, which become the basis for further inquiry. This can be carried out without much toil at first, some will be found at random, but entities with aptitude are likely to discover many of the fundamental laws influencing future stages.

At more advanced phases, joint action to solve new problems will make sense, and institutions to carry out this work will evolve. Some specialising in analysis, others in approximations would be plausible and congruent with the process under consideration. At a certain stage the usual analysis and approximations will not be enough. This could at first be noticed by the fact that continuous effort supply sometimes is insufficient, as some discoveries are only made when aided by chance with conventional methods. Thus, the craft of guessing will also be subject to specialisation.

It is understood that terms like *society* may be a subset of a greater system or have components regarded as one in line with Proposition 1, and not meant to disregard conflict of interest. This remark has far-reaching implications in conjunction with computational limitations. Which problem to solve is inherently a political project as it has the potential to disrupt status quo in favour of a class of players, and it precedes the politics of which solution to choose. The rules for this politics is system design, and as previous simple examples indicate, such design may evolve.

The development of risk management is another response, perhaps most concretely characterised by solution concepts which make use of mixed strategies as a best reply to uncertainty. This includes collective guessing which creates a chain reaction on a graph of problems, as the information from solved ones is applied to solve the remaining, thus increasing collective knowledge for future use. The understanding of the nature of these problems is open to some interpretation in applications, and not restricted to set of ideas. For instance, problem-solving can be taken to mean interaction, in the abstract, labour needed for potential social relations to be realised. In this manner the devices to master strategic interaction are produced by means of strategic interaction as a best reply in the work process. Problems which cannot be taken care of by the usual methods are rounded up for special treatment, to be taken care of collectively with information from neighbours, in a mixture including experience from previous successes, and randomness ensuing experimentation.

In summary, given a set of equilibrium practices, the choices among these possibly is a subject to play. Hence, there are incentives to erect institutions which deal with distribution given such a set. Moreover, it is possible to socialise with mixed equilibrium concepts given a set of equilibrium practices accordingly. Similar arrangements are conceivable in games about which problems to solve, and amounts to dealing with futures. Theoretical results predicting the characteristics of solutions, in line with

the results in paper 1, are essential here as they can be incorporated in the games of futures. In particular, if someone is endowed with the ability to predict the number of possible equilibria then such entity could benefit from the ignorance of the rest by selling rights to worthless prospects; or inferring when investments are reasonable given the amount of problems and impending advances. Ultimately one could conclude with certainty when all solutions have been found in larger games, without attempting to solve all problems. Artificial intelligence and advanced math are justified here, especially in view of patterns of problem difficulty and the conditionals inherent in Hypergeometrical distributions. In particular, theories with implications about who stands to gain from new advances would advance this project in multiple meanings. This emerges because although the concept of Nash Equilibria in a sense can be understood as outcomes from perfectly rational players who foresee all contingencies, these contingencies are given the plays of others, given an incentive structure and beliefs that others are inclined to behave in accordance to a scenario. What happens between these scenarios or if all the scenarios have not been computed is in turn to be understood as another game, or at least it is allowed.

Revolutionary Dynamics is compatible with top-down views of the world when it comes to modelling, in particular System Dynamics approaches. One way is just a matter of arranging a game between Leviathans as already outlined. Each experiment accommodates such systems, which then can invoke all the continuous dynamic needed, as each such experiment is not restricted to represent time. At one extreme, experiments may be taken to be simultaneous, thus defining location; at the other they are evolving stepwise; and in an intermediate case, the teleological features of the whole are disrupted by distributions which nevertheless can contain classes of congruence with one or the other extremes measured by the manner of distortion. Clearly, several parallel scenarios are allowed to evolve, and players may be concerned with a cluster of these cross-sectional *sequences* in decision-making.

What this adds to top-down modelling is that agency exposes power and its machinations, relevant to e.g. discuss Macro Economics; and perspectives on computational limitations which is relevant in the study of organisations. Agency also alters existing patterns. Games underpinning the postulated top-down functions are feasible, thus not only allowing feedback determined by the machines of the Leviathans, but allowing the machines themselves to transform, perhaps evolve in view of existing class relations and the politics of histories and futures. In this manner, these objects become subject to rules akin to objective laws, instead of merely relying on the petrified intuitions of a researcher.

With reference to paper 1, Replicator Dynamics can be played at an experiment, but be upset by an expansion of the game with new equilibria, which the players may end up being far away from; heterogeneity in myopic rationality is allowed. The mainstream way of questioning Economic doctrines with psychology and innate abilities is of course also available[6].

CLASSICAL ECONOMICS – DIALECTICS

Ultimately, this perspective also provides additional complementary tools to Philosophise and conduct thought experiments, which I think many may regard as far more important. An examination of one of the most influential and simultaneously obscure passages in the history of Political Economy captures what generation of the game-theoretical world means:

Mankind thus inevitably sets itself only such tasks as it is able to solve, since closer examination will always show that the problem itself arises only when the material conditions for its solution are already present or at least in the course of formation. (Karl Marx, 1859)

[6] Please note that this is not a serious challenge, as explicit Social Darwinism is one distinct and likely outcome in view of orthodoxy. For all of its failings, and perhaps for the wrong reasons, mainstream Economics has not always related differences in outcome directly to biology. There has been a remarkable naivety among some progressive enthusiasts, but I can assure you that many of those promoting these ideas over the years are not.

This passage is at the heart of dialectical materialism and notoriously abstruse. However, various theoretically sound interpretations are made intelligible by Revolutionary Dynamics. The potential social relations or material conditions are represented by systems which must be processed one way or another for any society to digest them. The process of solving such can be thought of as an abstract interaction, to make interactions materialise which, depending on interpretation, simply can be seen as getting familiar with new practices, ideas or customs; arriving at new ones; or the labour spent doing so, including research. The capitalist system, and other systems before it, consist of certain sets of problems corresponding to particular sets of incentive structures, i.e. the social fabric. Social interaction is encoded and computed by such systems in different ways which relate to the specific mode of production.

When a solution has been computed, it belongs to one of the social relations[7] which will tend to persist, and thus characterise a given social system. Much as new problems can be solved on the basis of old ones, by means of collective action as outlined above, older relations are keys to new patterns of interaction. As already commented, different social relations are more or less naturally linked to the extent they share at least one common solution. Although systems of components thus far can be seen as societies with social relations realised by the particular notion of Nash Equilibrium, it is possible to consider different systems governed by different solutions concepts, which in turn interact with top-down systems.

But whatever definition in line with aforementioned is chosen, in the end, necessary time to solve a problem is a measure, but differs in relation to technology, and also potentially transforms that technology (the software) when cracked – as an update for future developments.

[7] Or forces of production. Expositions about the exact definition range from ideological doctrines to analytical Philosophy. The take here is that if the technology to manage the hardware to solve problems is a force of production, then the and-connective is to be used.

The hardware can in principle be simulated by software, which allows more degrees of freedom regarding how forces of production relate to social relations, especially with regard to necessary time.

Why Marx insisted in a certain type of continuity of progression is mind-boggling, but there are plausible explanations within this framework. The distinct modes of production he had in mind must have a limited capacity to deal with problems which advance the forces of production and social relations, and the problems faced were under the influence of the ruling class, older social relations (solved problems), and chance.

As a given incentive structure is connected to a certain set of problems, those solved trivially correspond to its material conditions. Computation limitation corresponds to progression by solving some of these within a particular time frame, where a stochastic top-down choice could be one crude way of modelling the numerous decisions and activity in society which leads to certain advances – to make what he had in mind more tangible in terms of a politics of computation. This would then correspond to generating some problems at random, and it would relate to social conditions, as outlined in this and previous section. Moreover, much like in real systems, not all solutions will be unique, and could be reduced (e.g. by elimination of dominated strategies) to be contemplated in the abstract. Finally a cluster of problems is readily understood as 'the problem', which relates to the equilibrium customs of a given mode of production in transition, and the proofs in part 1.

A technical condition to initiate new incentive structures/problems must be added to capture richer notions on the possibility of progressive stages of development. As already hinted above, this could be done in more or less subtle ways involving existing problems. System Dynamics is eligible to generate incentive structures with laws of motions which then are rewritten and evolve as a consequence of strategic interaction, updating class and other relations in the process which conditions such interaction.

Rigorous formulation of what is mentioned in this part are worthy applications, perhaps suitable as Appendix material in future footnotes now that the installation cost of developing and checking universal solvers is sunken, thus allowing more time to writing and theory.

SUMMARY AND CONCLUSIONS

Revolutionary dynamics is a formal framework which enables syntheses with Game Theory, and provides an alternative perspective on Economics which challenges mainstream doctrines. Syntheses with System Dynamics and related approaches combines strategic interaction with a formal top-down view on society that allows both actors and the overarching system to evolve. Criticised mainstream notions of dynamics fluctuating around equilibria, effectively teaching inevitability of status quo, are transcended by a perspective on how and why social relations transform, in a manner which can disrupt old equilibria and conventions.

Universal computation and computation limitations are inherent to the perspective as it departs from the construction of universal solvers, and such devices are employed both by an observer of a system, and actors within the systems under consideration; and the limits can be exaggerated to consider historical systems. Overcoming such limitations is not only a feature of this project, but of the systems considered with it.

The formal systems presented are rich enough to make sense of Dialectical Materialism, with applications from Engineering and Philosophy, to models of simpler biological systems for problem solving.

It is possible to find all solutions to larger games without solving all the problems, but theory in line with paper 1 is needed to be certain. Artificial intelligence and advanced mathematics have a meaningful role to play to find promising problems and solutions, or develop such a theory.

On the layout. *Anachronism* would be a fitting name, partly relating to functional old-school pedagogical material without commercial, with a tone that supposedly helps readers focus. I agree. Today it could also make some readers feel included, and others furious, which is splendid.

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APPENDIX

The war-of-attrition problem. Maximising w.r.t. effort (time), yields the following differential equation:

Algorithm's Differential Equation for Max

$$bc[t] - \int_t^{\infty} c[r]dr = 0 \qquad bc'[t] + c[t] = 0$$

PERFORMANCE

Current **gmix[]** handles Fig.1 data in 70s with 53 solutions with t=1/3. This is close to the predicted time by a model based on E2 – but with an

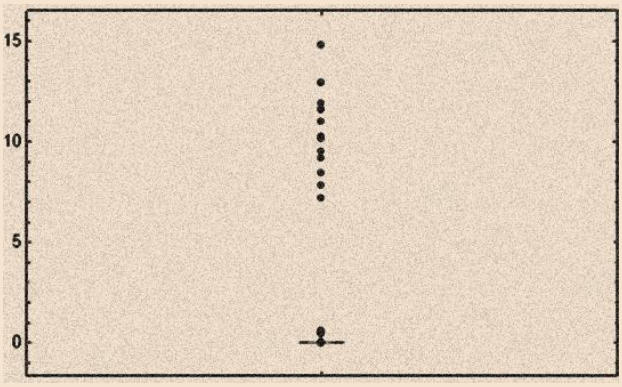
exponential approximation of problem difficulty and effort distribution as it is easy and fast to work with. Making use of monotonicity, and high density near origin; the requirements that the hardest third should be solved at least 50% of the time with a budget of 50s or less, and the least demanding third 995/1000, has the solution λ=6.92. This results in around 50 solutions around 55s, which is quite close to the expected value of 48s.

The following example makes use of data more closely corresponding to the terms of E2. The solution is λ=1.38 when 999/1000 should be solved for the simplest, 99/100 for the medium and 8/10 for the hard, within 68, 3, and 0.6 s. The random timer performs as the t= 1/3 but breaches the budget restriction as the approximation allows values above 1/3 63% of the time, with a mean about 0.7; and is moreover restricted to the same exponential distribution in all three as an example.

Additional explanations to technicalities and data.

Further descriptive statistics of the 2^7 game underscoring use of multiple timers. Max 14.8s; Min $3*10^{-4}$s; Median $4*10^{-3}$s; P(t>7|t>3/4) =1, while P(t <1/100|t<3/4) =82%.

FA1. Plot Effort



Thus T_A =3/4s, and T_N=15s works to give exact solutions to all except the 14 hard which are solved numerically with a 15s timer. The latter timer could be a more elaborated function of remaining time after analytical

solutions. In particular risk could be increased by dropping it completely and just attempt to solve the remaining problems one after one within global time budget T_G. Multiple timers in excess of two would still be relevant if there are preferences for a particular type of solution as a function of expected time requirements.

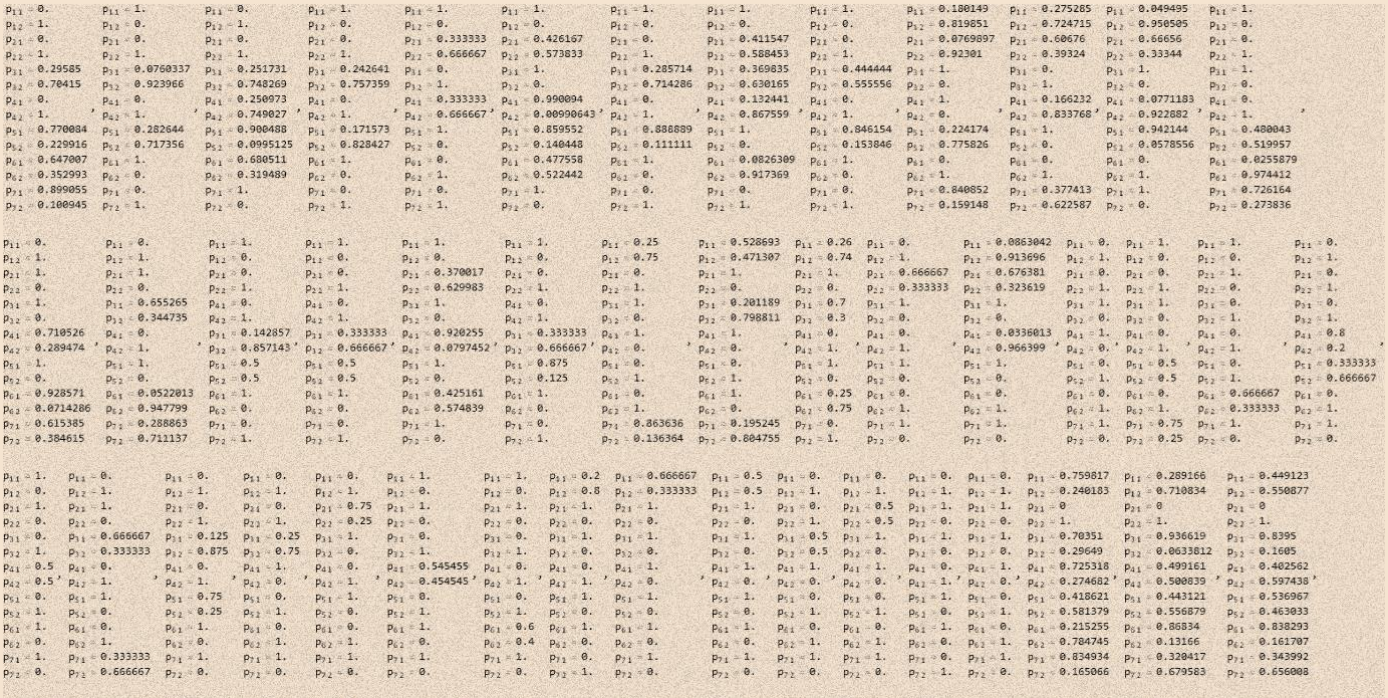
SOLUTIONS TO 7-PLAYER PROBLEMS

Solutions provided are particular, even when interval in general. A minor issue, and previous versions provided such info. Due to system issues and bug-search (see foreword on hacking), minor feats have not been cleared for research in latest update. Chain conjecture is prone to similar issues, addressed by extending the search in suspect region. **Solutions to additional problems provided on request**.

FIGURE 1

With reference to Appendix-section *Performance*. With analytical solver, t=1/3 contains 42 solutions, 3 with algebraic numbers. Presented as numerical approximations. To address the levels adjacent to the fully mixed, a switch to numerical solver yields 16 additional

TA1. Solution Set Fig1.

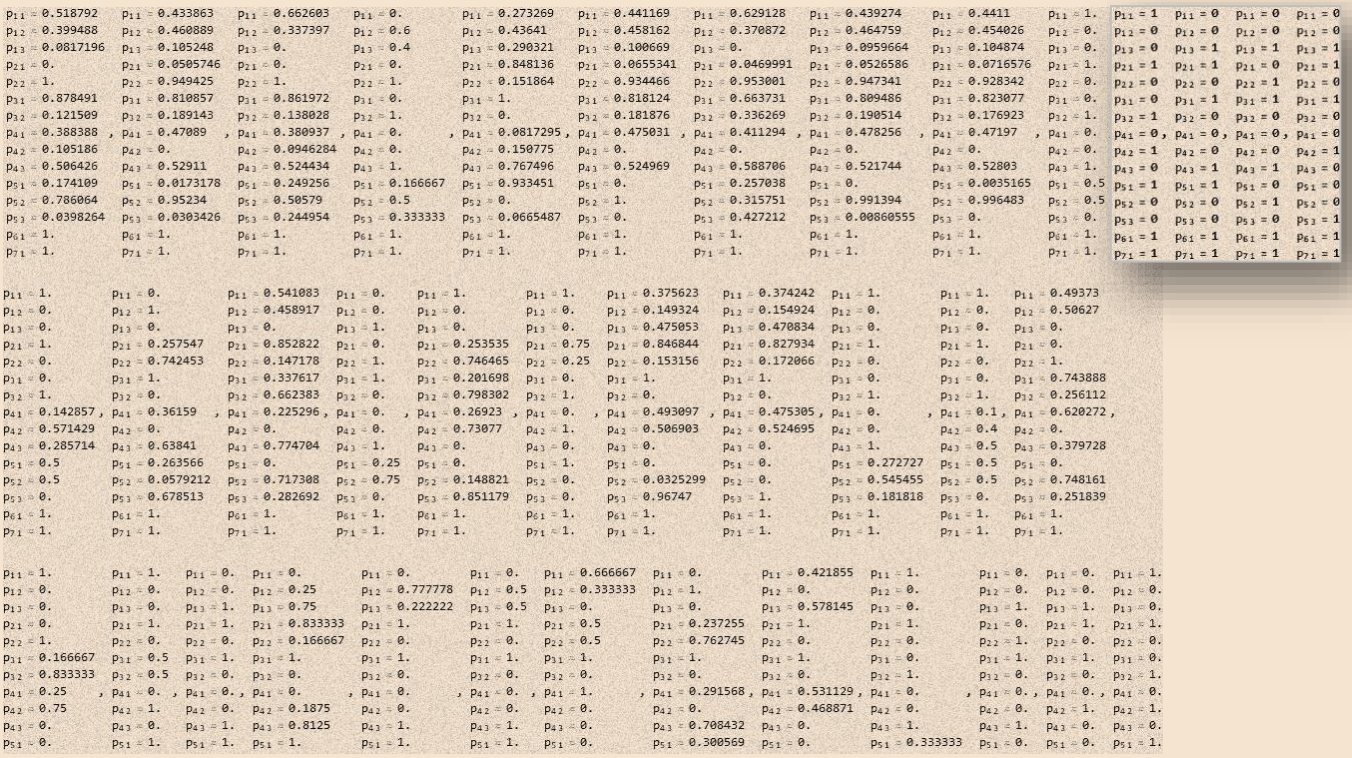


FA2 displays evolution of solutions starting from distributions when only one player randomises, to all but one at the far right. Note that a non-empty solution set is not guaranteed from this perspective. In this case, the completely randomised level is empty.

PROBLEM 1

34 solutions attained with **cc[]** based on a single analytical solver (10s). Waste about 2%[8]. For reference, the pure-strategy equilibria (see paper 1) are on a separate square to the right:

TA2. Solution Set Problem 1.



[8] Just the share of repeats. Note that this implicitly assumes a uniform time distribution. Measures in the vein of the latter that break this assumption are relevant, but depends on definitions and benchmarks in view of different solution techniques. More time could have been spent on the Fig.1 problem in this regard, but then again these results will be revised to allow information on interval solution anyway.